

SOME

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R E M A R K S

UPON THE

E Q U A T I O N O F T I M E,

AND THE TRUE

Manner of Computing it.

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ON THE

EVOLUTION OF TIME



Manner of Computing it

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SOME

REMARKS

UPON THE

EQUATION OF TIME,

AND THE

True Manner of Computing it.

THESE remarks were wrote above a twelve-month ago, and would have been then communicated to the Royal Society, had not my voyage to Barbados prevented it. Since my return from thence, I find part of the mistakes here pointed out acknowledged and corrected by M. Delalande, in his Treatise of Astronomy lately published, to whom I remember to have communicated my ideas on the subject, when he was in England. Nevertheless, as the error arising from

taking the equation of the equinoctial points into the account still remains uncorrected by him; and as I flatter myself, that what is said here may tend to set the whole matter in a clear light, I apprehend the publication of these remarks may still be proper.

The French Almanack, called the *Connoissance des Mouvements Celestes*, hath been deservedly esteemed by astronomers, as the most complete publication of its kind. Its present learned editor M. Delalande hath rendered its use more extensive by making the calculations from the latest and most approved tables, and also adding such explanations of them, as, at the same time, lay open before his readers the most considerable improvements of modern astronomy. Nevertheless, as the best mathematicians are not infallible, so I have reason to think I have discovered some errors in M. Delalande's method of computing the equation of time in this ephemeris, or, which comes to the same thing, the mean time, at the instant of apparent noon.

M. Delalande says, page of the *Connoissance* for 1760, which he repeats in the publications of other years, that, "to calculate exactly the difference between
 " mean and true time (that is to say the equation of
 " time) at the instant of apparent noon, the sum of
 " the equation of the sun's centre, the difference
 " between his longitude and right ascension, the lunar
 " equation, the equations of Jupiter and Venus, and
 " that of the precession of the equinoxes, with their
 " proper signs, must be converted into mean solar time.
 " He adds, that it was impossible, before this time,
 " to obtain the equation of time exactly; 1st, because
 " hitherto no account has been made of the four
 " little

“ little equations, the sum of which may produce
 “ above three seconds of time; 2dly, because it has
 “ been the practice to convert the equation of the
 “ sun’s centre, and the difference between his right
 “ ascension and longitude into time of the Primum
 “ Mobile, instead of converting them into mean
 “ solar time, which, says he, may produce an error
 “ of two seconds and a half; 3dly, because the
 “ equation of the sun’s centre was not known ex-
 “ actly before, every minute of which answers to
 “ four seconds in the equation of time.”

I readily agree with M. Delalande, that the equation of time could not be had so exactly formerly, as it may now, when we have a much more exact theory of the sun, and are lately made acquainted with new equations of his motion. I cannot, however, assent to his position, that the equation of the equinoctial points is to be taken into this account, together with the other equations, since this is not an inequality in the sun’s motion, but arises from a motion of the equator itself; yet of such a kind as cannot accelerate or retard the coming of the sun, or any star lying within the tropics, to the meridian, by above a quarter of a second of time. This will, perhaps, appear in a good measure plain, if it be considered, that the diurnal motion of the earth round its axis is neither accelerated nor retarded by the action of the sun and moon in producing the precession of the equinoxes, and variations of the inclination of the earth’s axis to the ecliptic. The effect of these actions is, that the terrestrial pole, each day, describes a small arc of a circle about the centre of the earth, in the plane of a celestial meridian passing through

through the sun or moon, or rather one between both; and, consequently, the equator of the earth has its motion in its own plane neither accelerated nor retarded, but obtains a new motion, whose axis is one of its own diameters. This is the true origin, as well of the minuter and periodical nutations, as of the regular and perpetual motion of the earth's axis about the pole of the ecliptic, observed in all ages, on which the continual precession of the equinoxes depends.

But, to illustrate more fully the point in question, let P, [see TAB. xx.] represent the north pole of the celestial equator, which suppose to be translated, in any certain time, from P to Q, through the small space P Q, upon the meridian P D, by the actions of the sun and moon; let A be the equinoctial point of aries, and S the sun or star. It is evident, that, as the rotation of the earth round its axis is no way affected, the translation of the celestial pole from P to Q along the arch P Q, of the celestial meridian P D, will occasion no alteration in the time of any given meridian of the earth coming to the fixt celestial meridian P D, nor consequently in the time of the sun or stars, when lying in this meridian, appearing to pass the meridian of the given place; contrary to what should follow from the method of computing the equation of time, used in the *Connoissance des Mouvements Celestes*; according to which, as long as the equation of the equinoxes is any thing, the equation of time must be affected thereby, and consequently the absolute time of the sun's passing the meridian.

But,

But, if the sun or star lie not in the celestial meridian $P D$, but in some other meridian $P S$, at S , then the spherical angle $\widehat{S P D}$ is the distance of the sun from the meridian $P D$, when the pole is at P , and $\widehat{S Q D}$ is his distance from the same meridian, when the pole is translated to Q . Let $P T$, $Q T$, meeting in T , be tangents of the meridians $P S$, $Q S$, in P and Q ; $\widehat{T Q D}$ being the external angle of the rectilineal triangle $T P Q$, the angle $\widehat{P T Q}$ is $= \widehat{T Q D} - \widehat{T P D} = \widehat{S Q D} - \widehat{S P D}$, and, therefore, is a measure of the alteration of the time of any meridian of the earth's coming to the sun at S , produced by the translation of the pole from P to Q . Now the sine of $\widehat{P T Q}$ is to the sine of $\widehat{T P Q}$, as $P Q$ to $T Q$; whence, calling the radius unity, and taking $P Q$ on account of its smallness, $=$ the sine of $P Q$, and the angle $\widehat{P T Q} =$ the sine of $\widehat{P T Q}$, we have $\widehat{P T Q} = \frac{P Q \times \text{sine } \widehat{T P Q}}{T Q} =$ the translation of the pole $\times$ the sine of the right ascension of the sun or star reckoned from the meridian in which the pole moves, $-$ the tangent of the polar distance, or, which is the same thing, $\times$ the tangent of the declination. Therefore, as $P Q$, arising from the nutation of the earth's axis, never exceeds $9''$, the greatest value of $\widehat{P T Q}$, for the sun can never exceed $9'' \times \text{tangent of } 23\frac{1}{2}^\circ$ the sun's greatest declination, $= 4''$, which answers to about $\frac{1}{2}$ of a second of time: and so much, and no more, may the sun come sooner or later to the meridian, on account of the nutation of the earth's axis: whereas, if the equation of the equinoxes was to be applied directly in the computation, according to

M. Delalande's

M. Delalande's method, it would sometimes, namely when at its maximum of $18''$, produce nearly $1\frac{1}{4}$ second of time.

But, tho' this demonstration may be admitted to be just, yet it may perhaps be asked, wherein lay the fault of the method of computation here censured, and whether the time of the sun's coming to the meridian is not regulated by his right ascension? It may also be thought requisite, that the true manner of computing the equation of time, from the sun's right ascension, should be shewn.

First, let it be observed, that when the pole is at P, A is the equinoctial point, and, when the pole is translated to Q, some other point B is the equinoctial point: therefore the sun's mean right ascension UPA is reckoned from A, and his apparent right ascension BQS, computed from his longitude, corrected by the equation of the equinoxes AB, or BS, is reckoned from another point B. Now the equation of time is proportional to the difference between the sun's mean and true right ascension, both reckoned from the same point; so that if the sun's mean right ascension is reckoned from A, his apparent right ascension, in this case, should be reckoned from A too; or if the apparent right ascension is reckoned, more properly, from the apparent right equinox B, his mean right ascension, for this purpose, should be reckoned from B likewise. For it is plain, from what has been said above, that no small motion of the pole P can at all affect the absolute time of a star in the equator's coming to the meridian of any place; for, the tangent QT then becoming infinite, the angle PTQ vanishes; therefore the mean equinox

nox A will come to the meridian at the same instant of absolute time, as if the pole had not been translated from P to Q; and the difference of time between the sun S coming to the meridian, and a fictitious sun U, supposed to move uniformly in the equator, with a motion equal to the sun's mean motion in longitude, or the equation of time will be therefore measured by $AQS \searrow APU$, the difference of their right ascensions reckoned from the same point A. It will also, by the like reasoning, be measured by $BQS \searrow BPU$, the difference of their right ascensions reckoned from the same point B; for B being the equinox, when the pole is at Q, the absolute time of the point B passing the meridian of any place will remain the same as if the pole had continued at P; whence the proposition easily follows, in like manner as above.

It may be now proper to shew how the equation of time, as affected by the nutation of the earth's axis, ought to be computed. This may be done two ways. The " first follows from what has been just
" laid down: correct the mean right ascension of
" the sun U P A, by the precession of the equinoxes
" in right ascension A P B (which is always to the
" precession in longitude B A, as cosine of the obli-
" quity of the ecliptic, to the radius, or as 12 to
" 13 nearly) the difference of the sun's mean right
" ascension thus corrected BPU, and the sun's appa-
" rent ascension BQS, turned into time, is the true
" equation of time."

Otherwise the effect of the nutation of the earth's axis upon the equation of time, is thought deserving notice, as it can never exceed $\frac{1}{2}$ of a second of time,

time, might be computed from the angle PTQ
 $= PQ \times \text{fine of } TPD \div TQ$, which, supposing
the nutation of the pole to be performed in a circle,
whose radius is $8''$, or a mean between the two con-
jugate semi-axes of the ellipsis, in which it really
moves, is $= 8'' \times \text{tangent of the sun's declination}$
 $\times \text{cosine of the difference of sun's right ascension,}$
and the longitude of the moon's ascending node.

But this is not the only mistake in the computa-
tion of the equation of time in the *Connoissance des*
Mouvements Celestes, though it may exceed one
second of time. M. Delalande says that the sum of
the equation of the sun's centre, the difference be-
tween his longitude and right ascension, and the
sum of the four little equations, must be converted
into mean solar time, in order to find the equation
of time; and adds, that no exact equation table
could be had, before this time, for three reasons,
one of which is, that it has always been the practice
to convert the equation of the sun's centre and the
difference between his longitude and right ascension
into time of the *Primum Mobile*, instead of mean
solar time, which, says he, may produce an error
of $2 \frac{1}{2}$ seconds.

Now I must here freely own, that as I could not,
without some reluctance, and only from the fullest
proof, allow all the mathematicians and astronomers,
before this time, to have been mistaken in the man-
ner of converting the quantities above-mentioned
into time, so I can find no reason to conclude so
from what has been cited above: on the contrary,
from a full consideration of the subject, I apprehend
the method hitherto used by the mathematicians to
be

be just, and that the author has himself fallen into an equal mistake with that of which he accuses them.

But, in order to set this matter in a clearer light, it will be first necessary to consider motion and time, relatively to each other; for, except this be done, it will be impossible to understand any thing precise from converting a certain number of minutes and seconds into mean solar time, or time of the *Primum Mobile*.

There are three different kinds of time used by astronomers, sidereal time, apparent solar time, and mean solar time. The interval between the transit of the first of Aries across the meridian one day, and its return to it the next day, is called a sidereal day, which is divided into 24 equal parts or hours, and the hours into minutes, &c. This time is shewn by a clock regulated to agree with the transit of the stars across the meridian. The interval between the transit of the sun across the meridian one day, and his transit the next day, is called an apparent solar day, which is divided into hours, minutes, &c. of apparent time. The solar day, it is manifest, and its hours, minutes, &c. are of different lengths, at different times of the year: on account of which inequality, a good clock, which keeps equal time, cannot long agree with the sun's motion, which is unequal. Therefore, astronomers have devised an imaginary time, called mean solar time; which is what would be pointed out by the sun, if his motion in right ascension from day to day was uniform, or, in other words, it is what would be pointed out by a fictitious sun or planet supposed to move uniformly in the equator, with a motion equal to the mean

motion of the sun in longitude, its distance from the first point of Aries (meaning hereby the mean equinox) being always equal to the mean longitude of the sun: and as apparent noon is the instant of the true sun's coming to the meridian, so mean noon is the instant at which this fictitious planet would come to the meridian. The interval between its coming to the meridian on any two successive days is a mean solar day, which is divided into hours, minutes, &c. of mean solar time; all which it is manifest will preserve the same length at all times of the year.

The equation of time, at the instant of apparent noon, or of the sun's passing the meridian, being equal to the difference between mean time and 12 hours, is also equal to the interval between the mean and true sun's passing the meridian expressed in mean solar time: to find which, we have the distance of the mean sun from the meridian, at the instant of apparent noon, equal to the difference between the sun's apparent and mean right ascension (both reckoned either from the mean or apparent equinox) which may be called the equation of right ascension. The question, therefore, comes to this, How many minutes and seconds of mean solar time doth the mean sun take to move this distance up to or from the meridian? Astronomers hitherto have allowed 1 minute of time to every 15 minutes of right ascension, and so in proportion; and, I apprehend, justly too; for does not the mean sun, in returning to the meridian, describe 360° about the pole in 24 hours of mean solar time; whence it is plain, that his departure from the meridian is at the rate of 15° to 1 hour, and $15'$ to 1 minute of mean solar time.

Therefore

Therefore astronomers have not converted the equation of right ascension into time according to the motion of the Primum Mobile; for, the equation of time being mean solar time, and the motion of the Primum Mobile being compleated in 23 H. 56 M. 4 S. of mean solar time, therefore 15° motion of the Primum Mobile does not answer to 1 hour of mean solar time (though it does to 1 hour of sidereal time) but to the 24th part of 23 H. 56 M. 4 S. or 59 M. $50\frac{1}{2}$ S. And it appears, that the equation of time in the *Connoissance des Mouvements Celestes* has been computed in this manner, and the table in the 79th page of the *Connoissance* for 1761 has been made use of, entitled, "A table to convert into degrees the time of a clock regulated according to the mean motion of the sun." The degrees of this table are evidently degrees of the Primum Mobile, 1 hour of mean solar time giving $15^\circ 2' 27,8''$, which answers to the motion of the stars from the meridian, but not to the mean motion of the sun from thence, which is 15° to 1 hour of mean solar time: whence it appears, that this writer hath evidently fell into the mistake of taking motion or space of the Primum Mobile, instead of the mean motion of the sun from the meridian; an equal mistake to that of which he erroneously supposes former mathematicians to have been guilty, in computing the equation of time. So that the equation of time in this ephemeris, besides the mistake arising from the taking in the equation of the equinoctial points into the account, is constantly too small in the proportion of 24 hours to 23 H. 56 M. 4 S. or of 366 to 365, or too small by 1 second upon every 6 minutes of the equation

equation of time : and the mistake of $2\frac{1}{2}$ seconds, which was supposed to be found in the old manner of reducing the equation of right ascension into time, really takes place in this new method ; which, added to 1 second of time, arising from the mistake in taking the precession of the equinoxes into the account, produces $3\frac{1}{2}$ seconds, an error which, I apprehend, the astronomical equation tables used since Mr. Flamsteed's time have but rarely exceeded.

To some, who are not well acquainted with the present improved state of astronomy, the difference in question may seem a matter of indifference, and too trifling for notice. But, if truth is the object of all our enquiries, why should we wilfully go beside it in the smallest matters ? And is it not a justice due to past astronomers, to whom we owe the foundations of all our knowledge, to vindicate them even from the smallest censure, which they do not appear to deserve ? At the same time, I flatter myself, that the learned editor of the *Connoissance des Mouvements Celestes*, and also the friends of the late illustrious Abbé de la Caille, who, I believe, was inadvertently the first author of this mistake, will take no offence at my endeavouring to clear up a point, which they, doubtless for want of having given sufficient attention to, seem to have mistaken : since, truth being the common object of all our pursuits, we ought candidly to accept as well the assistance we receive from each other for bringing us into the right road, when we happen to have strayed from it, as for helping us forward on our journey.

T H E E N D.



